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A METHOD FOR CALIBRATING A CONE-PROBE FLOW FIELD MEASURING INSTRUMENT

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ARO, Inc.

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ROCKET TEST FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
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Equation (34) should be as follows:

$$\sigma_{10} = \frac{B_2A_1 - B_1A_2}{C_2A_1 - C_1A_2}$$

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FOREWORD

The work reported herein was done at the request of Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), under Program Element 62402F, Project 3012, Task 07.

The results of research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract No. F40600-69-C-0001. The research was conducted between July 1968 and July 1969 under ARO Project No. BD5924, and the manuscript was submitted for publication on August 11, 1969.

This technical report has been reviewed and is approved.

Forrest B. Smith, Jr.	Harry L. Maynard
Research Division	Colonel, USAF
Directorate of Plans and Technology	Director of Plans and Technology

ABSTRACT

A simple method is presented for calibrating a cone-probe type of flow field measuring instrument. The calibration procedure requires data from only four orientations of the cone-probe in a quantitatively unknown flow field. Only changes in orientation must be known. It is then possible to use these data to determine the Mach number, total pressure, and flow angle of the unknown flow field; the constants in calibration equations for pitch and yaw flow angles; and the average static pressure. The method is verified by comparing with calibration results obtained in a conventional manner.

CONTENTS

APPENDIX

Illustrations

NOMENCLATURE

A	Coefficient in static pressure, Eqs. (11), (12), (13), (14)
A_1	Coefficient in Eq. (25)

A_2	Coefficient in Eq. (26)
a	Coefficient in static pressure, Eqs. (7), (8), (9), (10)
B	Coefficient in static pressure, Eqs. (11), (12), (13), (14)
B_1	Coefficient in Eq. (25)
B_2	Coefficient in Eq. (26)
b	Coefficient in static pressure, Eqs. (7), (8), (9), (10)
C_1	Coefficient in Eq. (25)
C_2	Coefficient in Eq. (26)
Cp	Pressure coefficient, $\left(\frac{p - p_\infty}{q}\right)$
ΔCp_ϵ	Differential pressure coefficient, $(Cp_c - Cp_a)$
ΔCp_σ	Differential pressure coefficient, $(Cp_d - Cp_b)$
$(\Delta Cp_\epsilon)_c$	Corrected differential pressure coefficient
$(\Delta Cp_\sigma)_c$	Corrected differential pressure coefficient
E	Defined by Eq. (38)
F	Defined by Eq. (39)
G	Defined by Eq. (29)
H	Defined by Eq. (30)
J	Defined by Eq. (31)
K	Defined by Eq. (32)
P	Nondimensional static pressure
P_A	Average nondimensional static pressure, Eq. (35)
P_{A_0}	Average nondimensional static pressure, for zero, angle of attack relative to aerodynamic axis, Eq. (36)
ΔP_ϵ	Nondimensional pressure difference, $(P_c - P_a)$
ΔP_σ	Nondimensional pressure difference, $(P_d - P_b)$
p	Measured static pressure (Fig. 1)
p_{t_m}	Measured total pressure (Fig. 1)
q	Free-stream dynamic pressure
α	Flow angle relative to reference axis (Fig. 1)
ϵ	Flow pitch angle (Fig. 1)

ϵ_0	Pitch angle of calibration flow field relative to reference coordinate in experiment I
ϵ_1	Pitch angle of reference axis in experiment III relative to its location in experiment I
ϵ_{10}	Pitch angle of aerodynamic axis relative to reference axis
θ	Roll angle (Fig. 1)
σ	Flow yaw angle (Fig. 1)
σ_0	Yaw angle of calibration flow field relative to reference coordinate in experiment I
σ_{10}	Yaw angle of aerodynamic axis relative to reference axis

SUBSCRIPTS

a, b, c, d	Static pressure taps
1, 2, 3, 4	Calibration experiments
∞	Free stream

SECTION I
INTRODUCTION

The cone-probe is an instrument for the absolute measurement of Mach number, total pressure, and flow angle. It consists of a cone-cylinder having four static pressure taps on the conical surface and a total pressure tap on the centerline. Applying both exact cone flow theory and linearized aerodynamic theory to the five pressure measurements yields Mach number, total pressure, and flow angle. In practice fabrication errors in combination with pressure measurement errors negate the simple application of aerodynamic theory and necessitate that the probe be experimentally calibrated.

A typical calibration (Ref. 1) consists of testing the probe in a known flow field at various known orientations. The resulting data are then used to construct calibration charts which relate (1) nondimensional static pressure differences to pitch and yaw angles of the flow field, (2) total pressure to pitch and yaw angle, and (3) nondimensional average static pressure to free-stream Mach number and the pitch and yaw angles of the flow. Some of the disadvantages of this type of calibration are

1. A large amount of experimental data is required.
2. The results are in graphical form and must be converted for computer use.
3. The angle of the flow relative to the centerline of the cone-probe must be accurately known for each experimental point.
4. The probe must be calibrated over the range of Mach numbers that the probe is expected to encounter when used.
5. The calibration flow field must be known.

In this report, a new calibration procedure is presented which requires only four experimental data points corresponding to four orientations of the cone-probe. These four experimental points are obtained by testing the probe in a flow field of unknown Mach number and flow angle. The four experimental points are used to determine (1) constants in calibration equations which relate pitch and yaw flow angles to nondimensional pressure differences, (2) constants in a calibration equation which relates a nondimensional average static pressure to the pitch and yaw angles of the flow, and (3) the Mach number, total pressure, and flow angles of the unknown flow field. The form of the calibration equations is established by the application of linearized aerodynamic theory to a generalized cone-probe geometry.

SECTION II
THEORY OF CALIBRATION

The theory of calibration presented in this report can be applied in a number of different ways, depending on the assumptions concerning the calibration flow field and the design of the cone-probe. In this report, the theory is applied, based on the following assumptions:

1. The calibration flow field is unknown.
2. The average static pressure obtained with the probe perfectly aligned with the flow can be theoretically related to the free-stream Mach number.
3. Static pressure readings from each tap vary linearly with flow angles up to 20 deg.
4. The measured total pressure is independent of flow angle up to 20 deg.

Assumptions 2, 3, and 4 are experimentally verified in Ref. 1.

Within the framework of these assumptions, only the effects of an angular misalignment of the static pressure taps need to be determined by a calibration. Such a calibration procedure is developed in the following sections.

SECTION III
BASIC CALIBRATION EQUATIONS

3.1 DERIVATION OF PITCH AND YAW CALIBRATION EQUATIONS

Each static pressure tap is assumed to be radially located at angles θ_a, θ_b, θ_c, and θ_d relative to an arbitrarily selected cylindrical coordinate system as shown in Fig. 1a (Appendix). The axis of this cylindrical coordinate system does not necessarily correspond to the physical centerline of the cone-probe. The calibration procedure determines the orientation of an "aerodynamic" axis which is defined to be the flow direction required to equalize the four static pressure measurements. For a perfectly constructed cone-probe, the aerodynamic axis coincides with the physical centerline of the cone-probe.

From Fig. 1b, the following trignometric relations can be derived:

$$\tan \epsilon = \tan \alpha \cos \theta \tag{1}$$
$$\tan \sigma = \tan \alpha \sin \theta \tag{2}$$
$$\tan^2 \epsilon + \tan^2 \sigma = \tan^2 \alpha \tag{3}$$

For flow angles less than 20 deg, Eqs. (1), (2), and (3) can be approximated as follows:

$$\epsilon = \alpha \cos \theta \tag{4}$$
$$\sigma = \alpha \sin \theta \tag{5}$$
$$\epsilon^2 + \sigma^2 = \alpha^2 \tag{6}$$

According to linearized aerodynamic theory (assumption 3), the static pressure at each tap can be represented by the following equations:

$$p_a = b_a + a_a \alpha \cos (\theta + \theta_a) \tag{7}$$
$$p_b = b_b + a_b \alpha \cos (\theta + \theta_b) \tag{8}$$
$$p_c = b_c + a_c \alpha \cos (\theta + \theta_c) \tag{9}$$
$$p_d = b_d - a_d \alpha \cos (\theta + \theta_d) \tag{10}$$

It is usual to nondimensionalize static pressure with the free-stream dynamic pressure; however, in this calibration, the free-stream dynamic pressure is unknown because the flow field is unknown (assumption 1). Therefore, the static pressures are nondimensionalized with the measured total pressure (p_{t_m}). Let

$$P_a = \frac{p_a}{p_{t_m}} \qquad P_c = \frac{p_c}{p_{t_m}}$$
$$P_b = \frac{p_b}{p_{t_m}} \qquad P_d = \frac{p_d}{p_{t_m}}$$
$$A_a = \frac{a_a}{p_{t_m}} \qquad B_a = \frac{b_a}{p_{t_m}}$$
$$A_b = \frac{a_b}{p_{t_m}} \qquad B_b = \frac{b_b}{p_{t_m}}$$
$$A_c = \frac{a_c}{p_{t_m}} \qquad B_c = \frac{b_c}{p_{t_m}}$$
$$A_d = \frac{a_d}{p_{t_m}} \qquad B_d = \frac{b_d}{p_{t_m}}$$

Substituting into Eqs. (7), (8), (9), and (10) yields

$$P_a = B_a + A_a\ \alpha\ \cos\ (\theta + \theta_a) \quad (11)$$
$$P_b = B_b + A_b\ \alpha\ \cos\ (\theta + \theta_b) \quad (12)$$
$$P_c = B_c + A_c\ \alpha\ \cos\ (\theta + \theta_c) \quad (13)$$
$$P_d = B_d + A_d\ \alpha\ \cos\ (\theta + \theta_d) \quad (14)$$

The constants A and B in each of Eqs. (11), (12), (13), and (14) are independent of α and θ. This is experimentally verified in Ref. 1.

Equations (11), (12), (13), and (14) can be expanded into the following forms:

$$P_a = B_a + A_a\ \alpha\ [\cos\theta\cos\theta_a - \sin\theta\sin\theta_a] \quad (15)$$
$$P_b = B_b + A_b\ \alpha\ [\cos\theta\cos\theta_b - \sin\theta\sin\theta_b] \quad (16)$$
$$P_c = B_c + A_c\ \alpha\ [\cos\theta\cos\theta_c - \sin\theta\sin\theta_c] \quad (17)$$
$$P_d = B_d + A_d\ \alpha\ [\cos\theta\cos\theta_d - \sin\theta\sin\theta_d] \quad (18)$$

Define

$$\Delta P_\epsilon = P_c - P_a \quad (19)$$

and

$$\Delta P_\sigma = P_d - P_b \quad (20)$$

Substituting Eqs. (15), (16), (17), and (18) into Eqs. (19) and (20) yields

$$\Delta P_\epsilon = (B_c - B_a) + \alpha\{[A_c\cos\theta_c - A_a\cos\theta_a]\cos\theta - [A_c\sin\theta_c - A_a\sin\theta_a]\sin\theta\} \quad (21)$$

and

$$\Delta P_\sigma = (B_d - B_b) + \alpha\{[A_d\cos\theta_d - A_b\cos\theta_b]\cos\theta - [A_d\sin\theta_d - A_b\sin\theta_b]\sin\theta\} \quad (22)$$

Equations (21) and (22) can be written in terms of ϵ and σ by substituting Eqs. (4) and (5). This yields

$$\Delta P_\epsilon = (B_c - B_a) + [A_c\cos\theta_c - A_a\cos\theta_a]\epsilon - [A_c\sin\theta_c - A_a\sin\theta_a]\sigma \quad (23)$$

and

$$\Delta P_\sigma = (B_d - B_b) + [A_d\cos\theta_d - A_b\cos\theta_b]\epsilon - [A_d\sin\theta_d - A_b\sin\theta_b]\sigma \quad (24)$$

Let

$$B_1 = (B_c - B_a) \qquad B_2 = (B_d - B_b)$$
$$A_1 = [A_c \cos\theta_c - A_a \cos\theta_a] \qquad A_2 = [A_d \cos\theta_d - A_b \cos\theta_b]$$
$$C_1 = [A_c \sin\theta_c - A_a \sin\theta_a] \qquad C_2 = [A_d \sin\theta_d - A_b \sin\theta_b]$$

Substituting into Eqs. (23) and (24) yields

$$\Delta P_\epsilon = B_1 + A_1\epsilon - C_1\sigma \tag{25}$$

$$\Delta P_\sigma = B_2 + A_2\epsilon - C_2\sigma \tag{26}$$

Solving for ϵ and σ yields

$$\epsilon = J(\Delta P_\sigma) - K(\Delta P_\epsilon) + \epsilon_{10} \tag{27}$$

and

$$\sigma = G(\Delta P_\epsilon) - H(\Delta P_\sigma) + \sigma_{10} \tag{28}$$

where

$$G = \frac{A_2}{C_2A_1 - C_1A_2} \tag{29}$$

$$H = \frac{A_1}{C_2A_1 - C_1A_2} \tag{30}$$

$$J = -H\left(\frac{C_1}{A_1}\right) \tag{31}$$

$$K = -G\left(\frac{C_2}{A_2}\right) \tag{32}$$

$$\epsilon_{10} = \frac{B_2C_1 - B_1C_2}{C_2A_1 - C_1A_2} \tag{33}$$

$$\sigma_{10} = \frac{B_2C_1 - B_1A_2}{C_2A_1 - C_1A_2} \tag{34}$$

The angles ϵ_{10} and σ_{10} are the pitch and yaw angles of the aerodynamic axis relative to the reference axis.

Equations (27) and (28) are the calibration equations for pitch (ϵ) and yaw (σ). Each equation involves three unknowns; and since the calibration flow field is unknown, there is a total of four unknowns (J, K, ϵ_{10}, and ϵ_0 for Eq. (27)). These four unknowns can be determined from four experiments.

3.2 DERIVATION OF CALIBRATION EQUATION FOR AVERAGE STATIC PRESSURE

The free-stream Mach number and total pressure are theoretically related to the ratio of average static pressure to total pressure. This relationship is based on the assumption that the average static pressure equals the surface static pressure on a cone at zero angle of attack. This is true only for a perfectly constructed cone-probe obeying linearized aerodynamic theory. For a practical cone-probe, the average static pressure is a function of flow angle as shown in the following derivation. Let

$$P_A = \frac{P_a + P_b + P_c + P_d}{4} \tag{35}$$

$$P_{A_0} = \frac{B_a + B_b + B_c + B_d}{4} \tag{36}$$

Substituting Eqs. (15), (16), (17), (18), and (36) into Eq. (35) yields

$$\begin{aligned} P_A = P_{A_0} &+ \frac{1}{4}[A_a \cos\theta_a + A_b \cos\theta_b + A_c \cos\theta_c + A_d \cos\theta_d]\, a \cos\theta \\ &- \frac{1}{4}[A_a \sin\theta_a + A_b \sin\theta_b + A_c \sin\theta_c + A_d \sin\theta_d]\, a \sin\theta \end{aligned} \tag{37}$$

Let

$$E = \frac{1}{4}[A_a \cos\theta_a + A_b \cos\theta_b + A_c \cos\theta_c + A_d \cos\theta_d] \tag{38}$$

and

$$F = \frac{1}{4}[A_a \sin\theta_a + A_b \sin\theta_b + A_c \sin\theta_c + A_d \sin\theta_d] \tag{39}$$

Substituting Eqs. (4), (5), (38), and (39) into Eq. (37) yields

$$P_A = P_{A_0} + E\epsilon - F\sigma \tag{40}$$

Equation (40) involves two unknowns (E and F); however, the calibration flow field is also unknown (assumption 1), so there is a total of three unknowns. These three unknowns can be determined using three of the four experiments required to determine the constants in flow angle calibration equations.

In Eq. (40), P_A is the measured nondimensional average static pressure, and P_{A0} is the nondimensional average static pressure for zero flow angle. Therefore, P_{A0} is used in the theory to determine free-stream Mach number and total pressure. From Eq. (40),

$$P_{A_0} = P_A - E\epsilon + F\sigma \tag{41}$$

SECTION IV
CALIBRATION EXPERIMENTS

Four experiments are required to determine all the unknowns in the calibration equations and the calibration flow field. Each experiment consists of a different orientation of the cone-probe with respect to the flow field. The orientations are arbitrary; however, relative orientation changes from experiment to experiment must be accurately known. The four experiments selected are discussed in the following sections.

4.1 EXPERIMENT I

The probe is located in the calibration flow field such that the flow angles are less than 20 deg relative to the reference coordinate system. The axis of the reference coordinate system is taken to be the geometric centerline of the cone-probe.

The unknown flow field is approaching this coordinate system at the angles ϵ_0 and σ_0. Therefore, in calibration Eqs. (25), (26), and (40), the following quantities are substituted:

$$\epsilon = \epsilon_0 \qquad \sigma = \sigma_0$$

$$\Delta P_\epsilon = (\Delta P_\epsilon)_1 \qquad \Delta P_\sigma = (\Delta P_\sigma)_1$$

$$P_A = (P_A)_1$$

Substituting into Eqs. (25) and (26) yields

$$(\Delta P_\epsilon)_1 = B_1 + A_1\epsilon_0 - C_1\sigma_0 \tag{42}$$

$$(\Delta P_\sigma)_1 = B_2 + A_2\epsilon_0 - C_2\sigma_0 \tag{43}$$

Substituting into Eq. (40) yields

$$(P_A)_1 = P_{A_0} + E\epsilon_0 - F\sigma_0 \quad . \tag{44}$$

4.2 EXPERIMENT II

From its position in experiment I, the cone-probe is rolled 90 deg clockwise about its axis, looking downstream. The quantities to be substituted into the calibration equations are

$$\epsilon = \sigma_0 \qquad \sigma = -\epsilon_0$$
$$\Delta P_\epsilon = (\Delta P_\epsilon)_2 \quad \Delta P_\sigma = (\Delta P_\sigma)_2 \quad P_A = (P_A)_2$$

Substituting into Eqs. (25) and (26) yields

$$(\Delta P_\epsilon)_2 = B_1 + A_1\sigma_0 + C_1\epsilon_0 \tag{45}$$

and

$$(\Delta P_\sigma)_2 = B_2 + A_2\sigma_0 + C_1\epsilon_0 \tag{46}$$

Substituting into Eq. (40) yields

$$(P_A)_2 = P_{A_0} + E\sigma_0 + F\epsilon_0 \tag{47}$$

4.3 EXPERIMENT III

The cone-probe is returned to its orientation in experiment I and then pitched to an angle, ϵ_1. The quantities to be substituted into the calibration equations are

$$\epsilon = \epsilon_0 + \epsilon_1 \qquad \sigma = \sigma_0$$
$$\Delta P_\epsilon = (\Delta P_\epsilon)_3 \qquad \Delta P_\sigma = (\Delta P_\sigma)_3$$
$$P_A = (P_A)_3$$

Substituting into Eqs. (25) and (26) yields

$$(\Delta P_\epsilon)_3 = B_1 + A_1(\epsilon_0 + \epsilon_1) - C_1\sigma_0 \tag{48}$$

and

$$(\Delta P_\sigma)_3 = B_2 - A_2(\epsilon_0 + \epsilon_1) - C_2\sigma_0 \qquad (49)$$

Substituting into Eq. (40) yields

$$(P_A)_3 = P_{A_0} + E(\epsilon_0 + \epsilon_1) - F\sigma_0 \qquad (50)$$

4.4 EXPERIMENT IV

From its position in experiment III, the cone-probe is rolled 90 deg clockwise, looking downstream. The quantities to be substituted in the calibration equations are

$$\epsilon = \sigma_0 \qquad \sigma = -(\epsilon_0 + \epsilon_1)$$
$$\Delta P_\epsilon = (\Delta P_\epsilon)_4 \qquad \Delta P_\sigma = (\Delta P_\sigma)_4$$

Substituting into Eqs. (25) and (26) yields

$$(\Delta P_\epsilon)_4 = B_1 + A_1\sigma_0 + C_1(\epsilon_0 + \epsilon_1) \qquad (51)$$

and

$$(\Delta P_\sigma)_4 = B_2 + A_2\sigma_0 + C_2(\epsilon_0 + \epsilon_1) \qquad (52)$$

There are now a sufficient number of equations to solve for the unknowns A_1, A_2, B_1, B_2, C_1, C_2, ϵ_0, σ_0, E, F, and P_{A_0}. These equations are summarized as follows:

$$(\Delta P_\epsilon)_1 = B_1 + A_1\epsilon_0 - C_1\sigma_0 \qquad (53)$$

$$(\Delta P_\epsilon)_2 = B_1 + A_1\sigma_0 + C_1\epsilon_0 \qquad (54)$$

$$(\Delta P_\epsilon)_3 = B_1 + A_1(\epsilon_0 + \epsilon_1) - C_1\sigma_0 \qquad (55)$$

$$(\Delta P_\epsilon)_4 = B_1 - A_1\sigma_0 + C_1(\epsilon_0 - \epsilon_1) \qquad (56)$$

$$(\Delta P_\sigma)_1 = B_2 + A_2\epsilon_0 - C_2\sigma_0 \qquad (57)$$

$$(\Delta P_\sigma)_2 = B_2 + A_2\sigma_0 + C_2\epsilon_0 \tag{58}$$

$$(\Delta P_\sigma)_3 = B_2 + A_2(\epsilon_0 + \epsilon_1) - C_2\sigma_0 \tag{59}$$

$$(\Delta P_\sigma)_4 = B_2 + A_2\sigma_0 + C_2(\epsilon_0 + \epsilon_1) \tag{60}$$

$$(P_A)_1 = P_{A_0} + E\epsilon_0 - F\sigma_0 \tag{61}$$

$$(P_A)_2 = P_{A_0} + E\sigma_0 + F\epsilon_0 \tag{62}$$

$$(P_A)_3 = P_{A_0} + E(\epsilon_0 + \epsilon_1) - F\sigma_0 \tag{63}$$

Solving for these unknowns yields

$$A_1 = \frac{(\Delta P_\epsilon)_3 - (\Delta P_\epsilon)_1}{\epsilon_1} \tag{64}$$

$$A_2 = \frac{(\Delta P_\sigma)_3 - (\Delta P_\sigma)_1}{\epsilon_1} \tag{65}$$

$$C_1 = \frac{(\Delta P_\epsilon)_4 - (\Delta P_\epsilon)_2}{\epsilon_1} \tag{66}$$

$$C_2 = \frac{(\Delta P_\sigma)_4 - (\Delta P_\sigma)_2}{\epsilon_1} \tag{67}$$

$$\sigma_0 = \frac{[(\Delta P_\sigma)_2 - (\Delta P_\sigma)_1]\,(C_1 - A_1) - [(\Delta P_\epsilon)_2 - (\Delta P_\epsilon)_1]\,(C_2 - A_2)}{2[A_2C_1 - A_1C_2]} \tag{68}$$

$$\epsilon_0 = \frac{(\Delta P_\sigma)_2 - (\Delta P_\sigma)_1 - (A_2 + C_2)\,\sigma_0}{(C_2 - A_2)} \tag{69}$$

$$B_1 = (\Delta P_\epsilon)_1 - A_1\epsilon_0 + C_1\sigma_0 \tag{70}$$

$$B_2 = (\Delta P_\sigma)_1 - A_2\epsilon_0 + C_2\sigma_0 \tag{71}$$

$$E = \frac{(P_A)_3 - (P_A)_1}{\epsilon_1} \tag{72}$$

$$P_{A_0} = \frac{(P_A)_1\,\epsilon_0 + (P_A)_2\,\sigma_0 - E(\epsilon_0{}^2 + \sigma_0{}^2)}{(\epsilon_0 + \sigma_0)} \tag{73}$$

$$F = \frac{(P_A)_2 - P_{A_0} - E\sigma_0}{\epsilon_0} \tag{74}$$

Most of the above equations are implicit in form and must be evaluated in the order in which they are presented.

The value of P_{A_0} obtained from Eq. (73) is used to determine the Mach number and total pressure of the calibration flow field. The quantity P_{A_0} is not a calibration constant in either Eq. (40) or (41).

The constants in calibration Eqs. (27) and (28) can now be determined from Eqs. (29), (30), (31), (32), (33), and (34).

SECTION V
MACH NUMBER EFFECTS

In practice, a cone-probe will encounter a range of Mach numbers in which the calibration equations [Eqs. (27), (28), and (41)] must be valid. It is, therefore, necessary to establish for these equations either an independence or their functional relationship with Mach number.

The linear form of the calibration equations can be assumed to be independent of Mach number; however, the range of flow angles in which they are acceptably valid is a function of Mach number.

The major constants in Eqs. (27) and (28) are K and H. These constants have theoretical values of -36.75 at a Mach number of 2.0 and -39.36 at a Mach number of 5.0 for a perfectly constructed 20-deg half-angle cone. Thus, these constants are a weak function of Mach number in the range from 2.0 to 5.0. This result is partially verified experimentally in Refs. 1 and 2 for the Mach number range from 1.51 to 3.51. The constants J and G are related to Mach number in the same manner as K and H and, therefore, are also a weak function of Mach number in the range from 2.0 to 5.0.

The constants E and F are zero for a perfectly constructed probe, independent of Mach number; however, the experimental data presented in Refs. 1 and 2 show a significant effect of Mach number on the relationship between average static pressure and flow angle.

SECTION VI
APPLICATION OF CALIBRATION PROCEDURE

To verify the calibration procedure, two sets of experimental data presented in Ref. 1 were analyzed.

The data were obtained in free streams having Mach numbers of 1.72 and 2.46. Unfortunately, the experimental data in Ref. 1 are presented in graphical form and nondimensionalized with respect to free-stream conditions. As a result, only the calibration equations for pitch and yaw angles could be obtained.

For a free-stream Mach number of 1.72, the following experimental data were obtained from Fig. 3 in Ref. 1.

Experiment I

To test the calibration procedure determination of the flow angle of the calibration flow field, experimental data for an angle of attack of 4.4 deg are used. The values are as follows:

$$(Cp_a)_1 = 0.295 \qquad (Cp_c)_1 = 0.468$$

$$(Cp_b)_1 = 0.350 \qquad (Cp_d)_1 = 0.365$$

For a Mach number of 1.72, $\dfrac{q}{p_{t_m}} = 0.4805$; therefore,

$$(\Delta P_\epsilon)_1 = [(Cp_c)_1 - (Cp_a)_1]\left(\frac{q}{p_{t_m}}\right) = 0.0831$$

$$(\Delta P_\sigma)_1 = [(Cp_d)_1 - (Cp_b)_1]\left(\frac{q}{p_{t_m}}\right) = 0.00721$$

$$(P_A)_1 = 0.403 \text{ (From Fig. 5, Ref. 1)}$$

Experiment II

The data for this experiment are the same as those for experiment I except shifted 90 deg.

$$(Cp_a)_2 = 0.350 \qquad (Cp_c)_2 = 0.365$$

$$(Cp_b)_2 = 0.468 \qquad (Cp_d)_2 = 0.295$$

and

$$(\Delta P_\epsilon)_2 = 0.00721$$
$$(\Delta P_\sigma)_2 = -0.0831$$
$$(P_A)_2 = 0.403$$

Experiment III

The data selected for this experiment are those obtained at an angle of attack of 9.4 deg. Therefore, ϵ_1 = 5 deg.

$$(Cp_a)_3 = 0.222 \qquad (Cp_c)_3 = 0.598$$
$$(Cp_b)_3 = 0.326 \qquad (Cp_d)_3 = 0.335$$

and

$$(\Delta P_\epsilon)_3 = 0.18075$$
$$(\Delta P_\sigma)_3 = 0.004325$$
$$(P_A)_3 = 0.403$$

Experiment IV

The data are as follows:

$$(Cp_a)_4 = 0.326 \qquad (Cp_c)_4 = 0.335$$
$$(Cp_b)_4 = 0.598 \qquad (Cp_d)_4 = 0.219$$

and

$$(\Delta P_\epsilon)_4 = 0.004325$$
$$(\Delta P_\sigma)_4 = -0.1821$$

Based on these four experiments, the calibration equations for the pitch and yaw angles are as follows:

$$\epsilon = 1.49\,(\Delta P_\sigma) + 51.12\,(\Delta P_\epsilon) - 0.03151 \text{ deg} \tag{75}$$
$$\sigma = 1.49\,(\Delta P_\epsilon) + 50.46\,(\Delta P_\sigma) - 0.02647 \text{ deg} \tag{76}$$

The pitch and yaw angles of the aerodynamic axis relative to the reference axis are -0.03151 and -0.02647 deg, respectively. The pitch and yaw angles of the calibration flow field relative to the reference axis are 4.20908 and 0.46125 deg, respectively. The pitch angle agrees well with the 4.4 deg given in Ref. 1 for the data used in experiment I.

The calibration plots for pitch and yaw angle presented in Ref. 1 are in terms of "corrected" pressure differences, nondimensionalized with respect to free-stream dynamic pressure. The corrected pressure differences are determined by the condition that the pitch and yaw angles are zero when the corrected pressure differences are zero. This neglects the misalignment of the aerodynamic axis with respect to the reference axis, and the measured flow angles are in error by the amount of this misalignment. The calibration equations [Eqs. (75) and (76)] corrected to satisfy these requirements are as follows:

$$\epsilon = -0.716\,(\Delta Cp_\sigma)_c + 24.55\,(\Delta Cp_\epsilon)_c \tag{77}$$

$$\sigma = 0.716\,(\Delta Cp_\epsilon)_c + 24.25\,(\Delta Cp_\sigma)_c \tag{78}$$

In Fig. 2, Eqs. (77) and (78) are graphically compared with the calibration curves of Ref. 1.

The second set of data from Ref. 1 were obtained at a free-stream Mach number of 2.46. The resulting calibration equations for pitch and yaw angles are as follows:

$$\epsilon = 1.122(\Delta P_\sigma) - 47.11(\Delta P_\epsilon) - 0.115435\text{ deg} \tag{79}$$

$$\sigma = -1.122(\Delta P_\epsilon) + 46.40(\Delta P_\sigma) - 0.113258\text{ deg} \tag{80}$$

or

$$\epsilon = 0.575(\Delta Cp_\sigma)_c + 24.13(\Delta Cp_\epsilon)_c \tag{81}$$

$$\sigma = -0.575(\Delta Cp_\epsilon)_c + 23.78(\Delta Cp_\sigma)_c \tag{82}$$

In Fig. 3, Eqs. (81) and (82) are graphically compared with the calibration curves of Ref. 1.

The comparisons presented in Figs. 2 and 3 clearly demonstrate the practical applicability of the calibration procedure developed in this report.

SECTION VII
CONCLUSIONS

The two major conclusions that can be drawn from this study of the cone-probe are as follows:

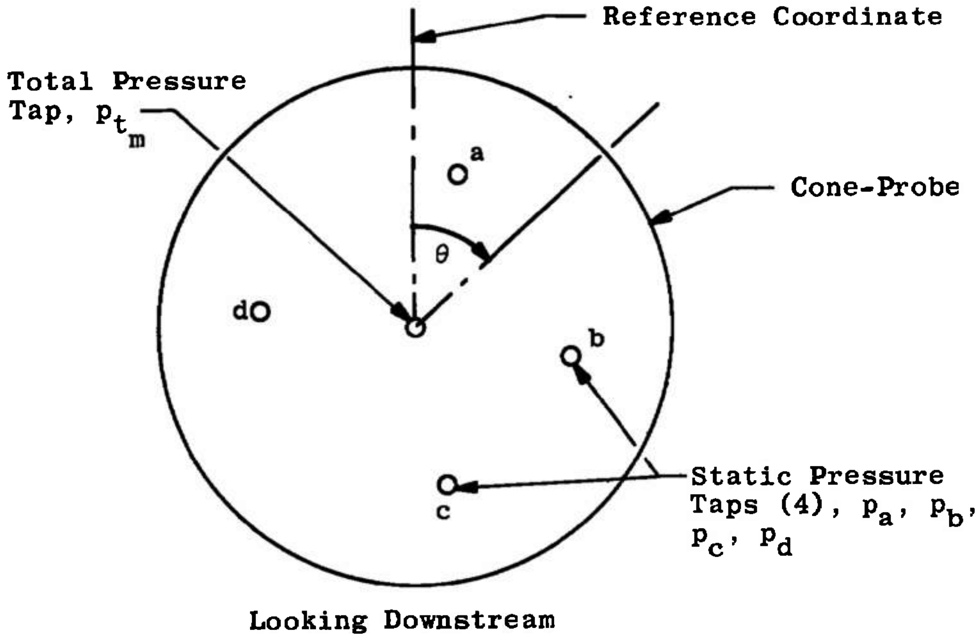
1. The qualitative aerodynamic behavior of a cone-probe is adequately described by linearized aerodynamic theory, and
2. Further work needs to be done to determine the influence of Mach number and Reynolds number on the calibration coefficients.

REFERENCES

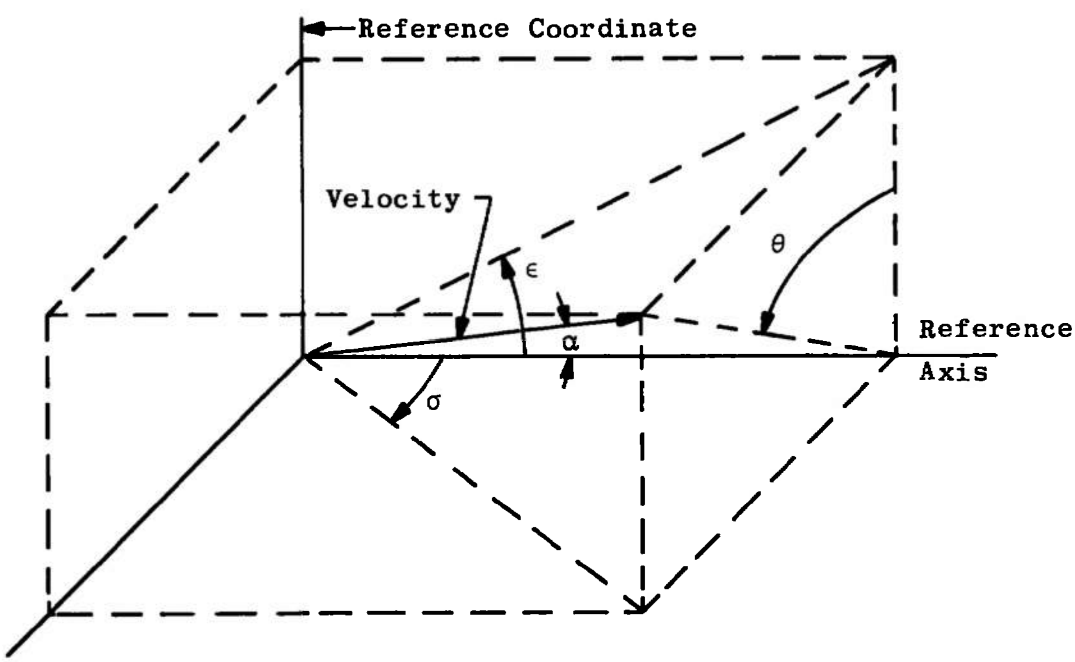
1. Centolanzi, F. J. "Characteristics of a 40° Cone for Measuring Mach Number, Total Pressure, and Flow Angles at Supersonic Speeds." NACA TN 3967, May 1957.

2. Vahl, W. A., and Weirich, R. L. "Calibration of 30° Included-Angle Cone for Determining Local Flow Conditions in Mach Number Range of 1.51 to 3.51." NASA TN D-4679, August 1968.

APPENDIX
ILLUSTRATIONS



a. Cone-Probe



b. Flow Field

Fig. 1 Cone-Probe and Flow Field Nomenclature

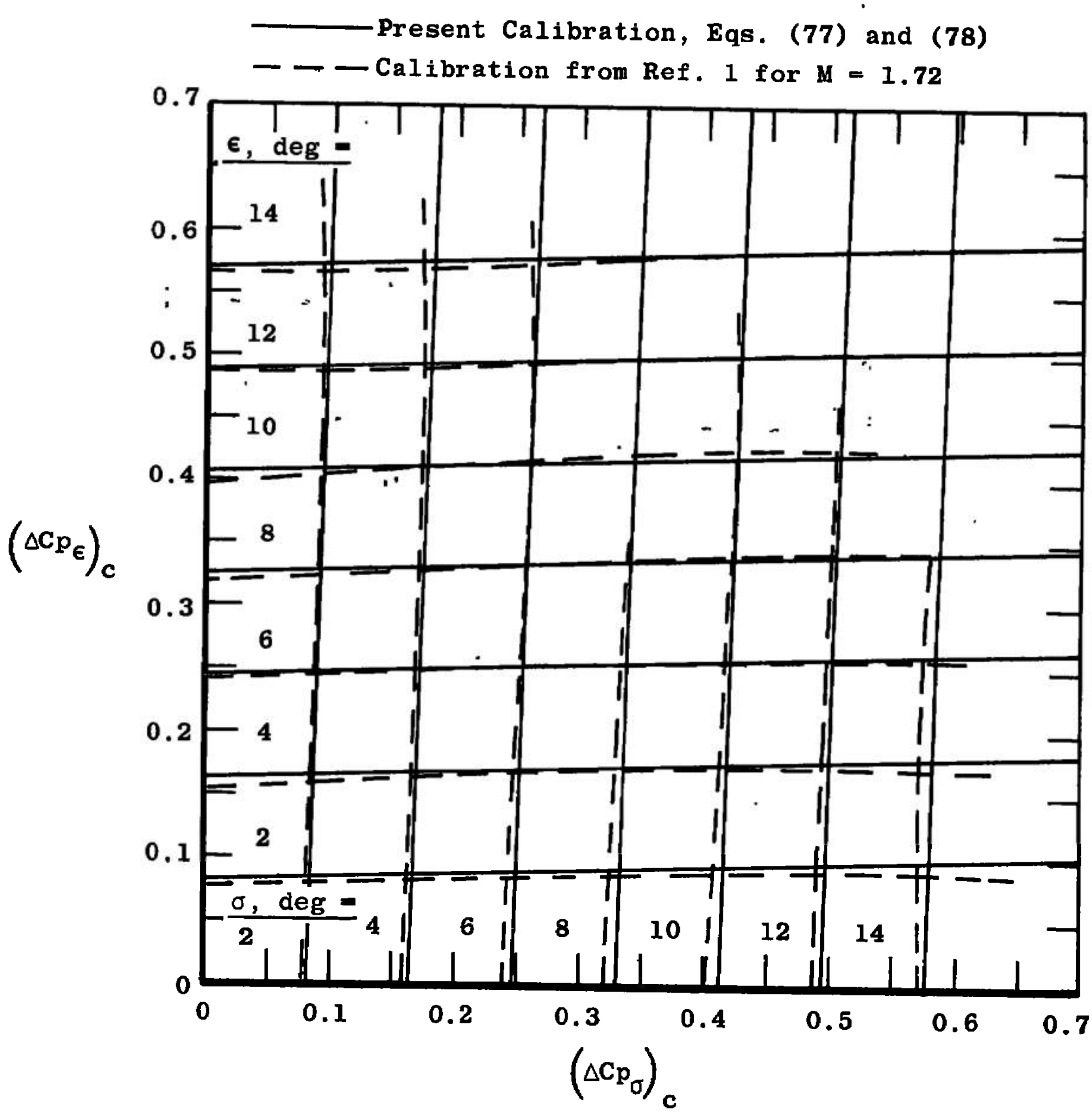


Fig. 2 Comparison of Calibration Curves For Mach Number 1.72

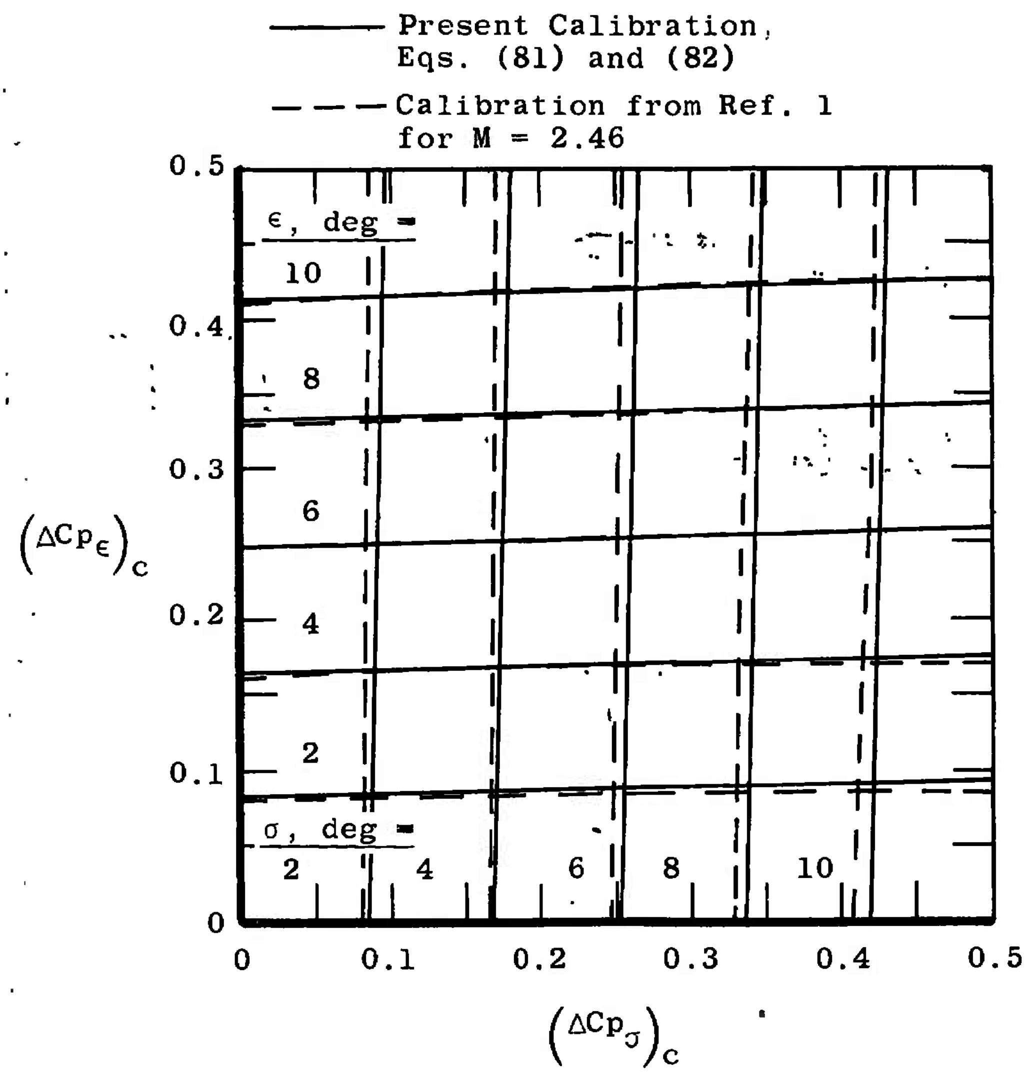


Fig. 3 Comparison of Calibration Curves for Mach Number 2.46

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13 ABSTRACT

A simple method is presented for calibrating a cone-probe type of flow field measuring instrument. The calibration procedure requires data from only four orientations of the cone-probe in a quantitatively unknown flow field. Only changes in orientation must be known. It is then possible to use these data to determine the Mach number, total pressure, and flow angle of the unknown flow field; the constants in calibration equations for pitch and yaw flow angles; and the average static pressure. The method is verified by comparing with calibration results obtained in a conventional manner.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
flow distribution						
flow measurement						
calibrating						
Mach number						
pressure						
measuring instruments						

1. Cone probes.

2 Probes -- Calibration

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